We call the following type the Curry–Howard interpretation of Brouwer's continuity principle

$$(\text{CH-Cont}) \qquad \Pi(f:\mathbb{N}^{\mathbb{N}}\to\mathbb{N})(\alpha:\mathbb{N}^{\mathbb{N}}).\ \Sigma(n:\mathbb{N}).\ \Pi(\beta:\mathbb{N}^{\mathbb{N}}).\ \alpha =_n \beta \to f(\alpha) = f(\beta).$$

Theorem. In intensional Martin-Löf type theory (with Π, Σ, Id),

$$\text{CH-Cont} \to 0 = 1.$$

Proof.

1. Assuming CH-Cont, we get a *modulus-of-continuity functional*
$$M : (\mathbb{N}^{\mathbb{N}} \to \mathbb{N}) \to \mathbb{N}$$
assigning a modulus $M(f)$ to f at the point 0^ω, where
   - 0^ω is the infinite sequence of zeros (`0`), and
   - $0^n k^\omega$ consists of n zeros followed by infinitely many k's (`n zeros-and-then k`).
   - Fact (i): $(0^n k^\omega)(n) = k$ ($\texttt{zeros-and-then-spec}_0$).
   - Fact (ii): $0^\omega =_n 0^n k^\omega$ ($\texttt{zeros-and-then-spec}_1$).
2. Let $m = M(\lambda\alpha.0)$. Define $f : \mathbb{N}^{\mathbb{N}} \to \mathbb{N}$ by $f(\beta) = M(\lambda\alpha.\beta(\alpha m))$.
3. By expanding the definitions (which involves the ξ-rule), we get
$$f(0^\omega) = M(\lambda\alpha.0^\omega(\alpha m)) = M(\lambda\alpha.0) = m \quad (\texttt{claim}_0).$$
4. By the definition of M, we have
$$\Pi(\beta : \mathbb{N}^{\mathbb{N}}).\ 0^\omega =_{M(f)} \beta \to m = f\beta \quad (\texttt{claim}_1).$$
5. Choosing $\beta = 0^{M(f)+1}1^\omega$, we have
$$0^\omega =_{M(f)} \beta \quad (\texttt{claim}_2)$$
and hence
$$m = f(\beta) \quad (\texttt{claim}_3).$$
6. By the continuity of $\lambda\alpha.\beta(\alpha m)$, we get
$$\Pi(\alpha : \mathbb{N}^{\mathbb{N}}).\ 0^\omega =_m \alpha \to \beta 0 = \beta(\alpha m) \quad (\texttt{claim}_4).$$
7. Choosing $\alpha = 0^m(M(f)+1)^\omega$, we have
$$0^\omega =_m \alpha \quad (\texttt{claim}_5)$$
and hence
$$0 = \beta 0 = \beta(\alpha m) = \beta(M(f)+1) = 1 \quad (\texttt{goal}).$$