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A Generic Type System for Featherweight Java

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GuideForce¹

GuideForce develops **effect type systems** for enforcing secure programming guidelines.

- Imagine that functions of interests emit **events** when they are executed.
 - E.g., `Server.login()` emits a *login* event; `Connection.close()` emits a *close* event; ...
 - Each execution of a program generates a (finite or infinite) trace of events.
 - Guidelines (of safety and liveness properties) specify which event traces are allowed.
- The type system has effect annotations to give information about the possible traces.
 - E.g., `login() ? readData() : close();` : **type** & {*login read*, *login close*}
- Inferring the type of a program is to compute its effect.
- If the effect is “contained” in the guideline, then the program adheres to the guideline.

¹ GuideForce (DFG 250888164) was Initiated by Martin Hofmann at LMU, and is now hosted at fortiss.

Region Typing



- ▶ If a method was analyzed without considering object information, then its effect should include the traces of all objects.
E.g., `y.last()` and `z.last()` would have the same effect.
- ▶ Then the terminating method `linear()` would have the same effect of the nonterminating method `cyclic()`.
- ▶ To improve the precision of effect typing, we use **regions** to narrow down referenced objects.
Objects in different regions are analyzed separately.

```
1  class Node {
2      Node next;
3      Node last() {
4          emit(a);
5          if (next == null) {
6              return this;
7          } else {
8              return next.last();
9          }
10     }
11 }
12
13 Class Test {
14     Node linear() {
15         Node x = new Node();
16         Node y = new Node();
17         y.next = x;
18         return y.last();
19     }
20     Node cyclic() {
21         Node z = new Node();
22         z.next = z;
23         return z.last();
24     }
25 }
```

Region Type Systems for Featherweight Java

► A pure region type system

[BGH13] L. Beringer, R. Grabowski, and M. Hofmann. **Verifying Pointer and String Analyses with Region Type Systems.** *Computer Languages, Systems & Structures* 39(2), 49–65, 2013.

► A region-based effect type system (for analyzing terminating behaviors)

[EHZ17] S. Erbatur, M. Hofmann, and E. Zălinescu. **Enforcing Programming Guidelines with Region Types and Effects.** *APLAS 2017*.

► Büchi effects (abstract interpretation based on Büchi automata)

[HC14] M. Hofmann and W. Chen. **Abstract Interpretation from Büchi Automata.** *CSL-LICS 2014*.

► Another region-based effect type system (nonterminating and exceptional behaviors)

[ESX21] S. Erbatur, U. Schöpp, and C. Xu. **Type-based Enforcement of Infinitary Trace Properties for Java.** To appear at PPDP 2021.

Goal: unify the above systems

- ❖ Understand the essential structure
- ❖ Relate and compare to other frameworks
- ❖ Avoid redundant work on the meta theory

Featherweight Java (FJ)

► Four kinds of names

variables: $x, y \in \text{Var}$ classes: $C, D \in \text{Cls}$ fields: $f \in \text{Fld}$ methods: $m \in \text{Mtd}$

► Special formal elements

$\text{this} \in \text{Var}$ $\text{Object}, \text{NullType} \in \text{Cls}$

► FJ expressions

$\text{Expr} \ni e ::= x \mid \text{let } x = e_1 \text{ in } e_2 \mid \text{if } x = y \text{ then } e_1 \text{ else } e_2 \mid \text{null} \mid \text{new}^\ell C \mid (C) e$
 $\mid x^C.f \mid x^C.f := y \mid x^C.m(\bar{y})$

► An FJ program ($<, \text{fields}, \text{methods}, \text{mtable}$) consists of

- a subtyping relation $< \in \mathcal{P}^{\text{fin}}(\text{Cls} \times \text{Cls})$
- a field list $\text{fields} : \text{Cls} \rightarrow \mathcal{P}^{\text{fin}}(\text{Fld})$
- a method list $\text{methods} : \text{Cls} \rightarrow \mathcal{P}^{\text{fin}}(\text{Mtd})$
- a method table $\text{mtable} : \text{Cls} \times \text{Mtd} \rightarrow \text{Var}^* \times \text{Expr}$

Example of an FJ program

► Java code

```
1  class Node {  
2      Node next;  
3      Node last() {  
4          emit(a);  
5          if (next == null) {  
6              return this;  
7          } else {  
8              return next.last();  
9          }  
10     }  
11 }
```

► FJ program

fields(Node) = {next}

methods(Node) = {last}

mtable(Node, last) = $((), e_{\text{last}})$

$e_{\text{last}} :=$ let $_ = \text{emit}(a)$ in

let $x = \text{this.next}$ in

let $y = \text{null}$ in

if $x = y$ then this

else let $z = \text{this.next}$ in $z.\text{last}()$

A Parametric Operational Semantics

► State model

locations: $l \in \text{Loc}$

stores: $s \in \text{Var} \rightarrow \text{Val}$

values: $v \in \text{Val} = \text{Loc} \uplus \{\text{null}\}$

heaps: $h \in \text{Loc} \rightarrow \text{Obj}$

objects: $(C, G, \ell) \in \text{Obj} = \text{Cls} \times (\text{Fld} \rightarrow \text{Val}) \times \text{Pos}$

Write \mathcal{V} to denote the set of pairs (v, h) of values and heaps.

► **Parameter:** a set \mathcal{M} together with functions

$\text{return}_{\mathcal{M}} : \mathcal{V} \rightarrow \mathcal{M}$

$\text{bind}_{\mathcal{M}} : \mathcal{M} \times \mathcal{M} \rightarrow \mathcal{M}$

$|-|_{\mathcal{M}} : \mathcal{M} \rightarrow \mathcal{V}$

such that

$|\text{return}_{\mathcal{M}}(v, h)|_{\mathcal{M}} = (v, h)$ and $|\text{bind}_{\mathcal{M}}(m_1, m_2)|_{\mathcal{M}} = |m_1|_{\mathcal{M}} \text{ or } |m_2|_{\mathcal{M}}$

► **Big-step relation** $(s, h) \vdash e \Downarrow m$

Intuition: In state (s, h) the expression e evaluates to the value v with the heap updated to h' , where $(v, h') = |m|_{\mathcal{M}}$.

Operational Semantics Rules

$$\frac{}{(s, h) \vdash x \Downarrow \text{return}_{\mathcal{M}}(s(x), h)} \quad \frac{}{(s, h) \vdash \text{null} \Downarrow \text{return}_{\mathcal{M}}(\text{null}, h)}$$

$$\frac{(s, h) \vdash e_1 \Downarrow m_1 \quad (v_1, h_1) = |m_1|_{\mathcal{M}} \quad (s[x \mapsto v_1], h_1) \vdash e_2 \Downarrow m_2}{(s, h) \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow \text{bind}_{\mathcal{M}}(m_1, m_2)}$$

$$\frac{s(x) = s(y) \quad (s, h) \vdash e_1 \Downarrow m}{(s, h) \vdash \text{if } x = y \text{ then } e_1 \text{ else } e_2 \Downarrow m} \quad \frac{s(x) \neq s(y) \quad (s, h) \vdash e_2 \Downarrow m}{(s, h) \vdash \text{if } x = y \text{ then } e_1 \text{ else } e_2 \Downarrow m}$$

$$\frac{l \notin \text{dom}(h) \quad G = [f \mapsto \text{null}]_{f \in \text{fields}(C)}}{(s, h) \vdash \text{new}^{\ell} C \Downarrow \text{return}_{\mathcal{M}}(l, h[l \mapsto (C, G, \ell)])}$$

$$\frac{(s, h) \vdash e \Downarrow m \quad (v, h') = |m|_{\mathcal{M}} \quad \text{classOf}_{h'}(v) \leq C}{(s, h) \vdash (C) e \Downarrow m}$$

$$\frac{s(x) = l \quad h(l) = (_, G, _)}{(s, h) \vdash x.f \Downarrow \text{return}_{\mathcal{M}}(G(f), h)} \quad \frac{s(x) = l \quad h(l) = (D, G, \ell) \quad h' = h[l \mapsto (D, G[f \mapsto s(y)], \ell)]}{(s, h) \vdash x.f := y \Downarrow \text{return}_{\mathcal{M}}(s(y), h')}$$

$$\frac{s(x) = l \quad h(l) = (D, _, _) \quad \text{mtable}(D, m) = (\bar{z}, e) \quad ([\text{this} \mapsto l] \cup [z_i \mapsto s(y_i)]_{i \in \{1, \dots, |\bar{z}|\}}, h) \vdash e \Downarrow m}{(s, h) \vdash x.m(\bar{y}) \Downarrow m}$$

Instances of the Operational Semantics

► Standard FJ operational semantics

Simply take $\mathcal{M} = \mathcal{V}$, and $\text{return}_{\mathcal{M}}$ and $|-|_{\mathcal{M}}$ the identity, and $\text{bind}_{\mathcal{M}}$ the second projection.

E.g.
$$\frac{(s, h) \vdash e_1 \Downarrow v_1, h_1 \quad (s[x \mapsto v_1], h_1) \vdash e_2 \Downarrow v_2, h_2}{(s, h) \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow v_2, h_2}$$

► Operational semantics with trace effects

Apply the writer monad $X \mapsto X \times \Sigma^*$, i.e., take $\mathcal{M} = \mathcal{V} \times \Sigma^*$ and

$$\begin{aligned} \text{return}_{\mathcal{M}}(v, h) &= ((v, h), \varepsilon) \\ \text{bind}_{\mathcal{M}} \left((_, w_1), ((v_2, h_2), w_2) \right) &= ((v_2, h_2), w_1 w_2) \\ |((v, h), _) |_{\mathcal{M}} &= (v, h) \end{aligned}$$

E.g.
$$\frac{(s, h) \vdash e_1 \Downarrow v_1, h_1 \ \& \ w_1 \quad (s[x \mapsto v_1], h_1) \vdash e_2 \Downarrow v_2, h_2 \ \& \ w_2}{(s, h) \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow v_2, h_2 \ \& \ w_1 w_2}$$

► Operational semantics for FJ extended with e.g. exceptions

Region Types

- ▶ A **region** represents a **property** of a value such as its provenance information.

$$\text{Reg} \ni r, s ::= \text{Null} \mid \text{CreatedAt}(\ell) \mid \top \mid \perp \mid r \vee s \mid r \wedge s$$

- ▶ A **formal interpretation** of regions as a relation $(v, h) \vdash r$

$$\frac{}{(null, h) \vdash \text{Null}} \quad \frac{h(l) = (C, G, \ell)}{(l, h) \vdash \text{CreatedAt}(\ell)} \quad \frac{}{(v, h) \vdash \top} \quad \frac{(v, h) \vdash r}{(v, h) \vdash r \vee s} \quad \frac{(v, h) \vdash s}{(v, h) \vdash r \vee s} \quad \frac{(v, h) \vdash r \quad (v, h) \vdash s}{(v, h) \vdash r \wedge s}$$

- ▶ The interpretation gives a **partial order** \leq on regions

$$r \leq s \text{ iff } (v, h) \vdash r \text{ implies } (v, h) \vdash s \text{ for all } (v, h) \in \mathcal{V}.$$

- ▶ Regions form a **lattice** $(\text{Reg}, \leq, \vee, \wedge)$

A Generic Type System for FJ

- **Parameter:** a join-semilattice $(\mathcal{L}, \emptyset, \sqsubseteq, \sqcup)$ together with function

$$\text{return}_{\mathcal{L}} : \text{Reg} \rightarrow \mathcal{L} \quad \text{bind}_{\mathcal{L}} : \mathcal{L} \times (\text{Reg} \rightarrow \mathcal{L}) \rightarrow \mathcal{L} \quad |{-}|_{\mathcal{L}} : \mathcal{L} \rightarrow \mathcal{P}(\text{Reg})$$

Idea: \mathcal{L} may carry information of e.g. regions, effects or probabilities with various representations.

The **essential structure** of a region type system for FJ is given by a monad on the region lattice.

- **Typing judgments** have the form $x_1:r_1, \dots, x_n:r_n \vdash e : T$ where $r_1 \in \text{Reg}$ and $T \in \mathcal{L}$.

- A **class table** (F, M) consists of

- a **field typing** $F : \text{Cls} \times \text{Reg} \times \text{Fld} \rightarrow \text{Reg}$, and
- a **method typing** $M : \text{Cls} \times \text{Reg} \times \text{Mtd} \times \text{Reg}^* \rightarrow \mathcal{L}$

satisfying some well-formedness conditions that reflect the subtyping properties of FJ.

- An FJ program is **well-typed** w.r.t. (F, M) if each method body has the type as specified in M ,
i.e. $\text{this}:r, \bar{x}:\bar{s} \vdash e : T$ holds for any (C, r, m, \bar{s}) with $M(C, r, m, \bar{s}) = T$ and $\text{mtable}(C, m) = (\bar{x}, e)$.

Typing Rules

$$\begin{array}{c}
 \text{BOT} \frac{(x: \perp) \in \Gamma}{\Gamma \vdash e : \emptyset} \qquad \text{SUB} \frac{\Gamma \vdash e : T \quad T \sqsubseteq T'}{\Gamma \vdash e : T'} \\
 \\
 \text{VAR} \frac{}{\Gamma, x:r \vdash x : \text{return}_{\mathcal{L}}(r)} \qquad \text{NULL} \frac{}{\Gamma \vdash \text{null} : \text{return}_{\mathcal{L}}(\text{Null})} \\
 \\
 \text{LET} \frac{\Gamma \vdash e_1 : T_1 \quad \Gamma, x:r \vdash e_2 : f(r) \text{ for all } r \in |T_1|_{\mathcal{L}}}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \text{bind}_{\mathcal{L}}(T_1, f)} \\
 \\
 \text{IF} \frac{\Gamma, x:r \wedge s, y:r \wedge s \vdash e_1 : T_1 \quad \Gamma, x:r, y:s \vdash e_2 : T_2}{\Gamma, x:r, y:s \vdash \text{if } x = y \text{ then } e_1 \text{ else } e_2 : T_1 \sqcup T_2} \\
 \\
 \text{NEW} \frac{}{\Gamma \vdash \text{new}^{\ell} C : \text{return}_{\mathcal{L}}(\text{CreatedAt}(\ell))} \qquad \text{CAST} \frac{\Gamma \vdash e : T}{\Gamma \vdash (D) e : T} \\
 \\
 \text{GET} \frac{s = F(C, r, f)}{\Gamma, x:r \vdash x^C.f : \text{return}_{\mathcal{L}}(s)} \qquad \text{SET} \frac{s \leq F(C, r, f)}{\Gamma, x:r, y:s \vdash x^C.f := y : \text{return}_{\mathcal{L}}(s)} \\
 \\
 \text{CALL} \frac{T = M(C, r, m, \bar{s})}{\Gamma, x:r, \bar{y}:\bar{s} \vdash x^C.m(\bar{y}) : T}
 \end{array}$$

A Uniform Soundness Theorem

- Lift $(v, h) \vdash r$ to typing environments Γ and field typing F : $(s, h) \vdash \Gamma, F$

It says that the **state** (for evaluating the program) **satisfies** the properties specified by **the typing**.

- **Last parameter** $\triangleleft \subseteq \mathcal{M} \times \mathcal{L}$ to **relate the parameters** \mathcal{M} and \mathcal{L}

Soundness Theorem. Suppose $\triangleleft \subseteq \mathcal{M} \times \mathcal{L}$ preserves the structures on \mathcal{M} and \mathcal{L} in the following sense:

($\triangleleft 1$) $m \triangleleft T$ and $T \sqsubseteq T'$ implies $m \triangleleft T'$,

($\triangleleft 2$) $(v, h) \vdash r$ implies $\text{return}_{\mathcal{M}}(v, h) \triangleleft \text{return}_{\mathcal{L}}(r)$, and

($\triangleleft 3$) if $m \triangleleft T$ and if $m' \triangleleft f(r)$ for all $r \in |T|_{\mathcal{L}}$ with $|m|_{\mathcal{M}} \vdash r$, then $\text{bind}_{\mathcal{M}}(m, m') \triangleleft \text{bind}_{\mathcal{L}}(T, f)$.

Given an FJ program that is well-type w.r.t. (F, M) , for any s, h, e, m, Γ and T such that

$$(s, h) \vdash e \Downarrow m \quad \text{and} \quad \Gamma \vdash e : T \quad \text{and} \quad (s, h) \vdash \Gamma, F$$

we have $m \triangleleft T$ and $(s, h') \vdash \Gamma, F$ where $(_, h') = |m|_{\mathcal{M}}$.

Instantiating the Type System

- ▶ To build a concrete type system,
provide a join-semilattice $(\mathcal{L}, \emptyset, \sqsubseteq, \sqcup)$ with maps $\text{return}_{\mathcal{L}}$, $\text{bind}_{\mathcal{L}}$ and $|-|_{\mathcal{L}}$.
- ▶ To establish its soundness result,
 - instantiate the operational semantics, i.e., choosing a set \mathcal{M} with maps $\text{return}_{\mathcal{M}}$, $\text{bind}_{\mathcal{M}}$ and $|-|_{\mathcal{M}}$
 - specify the relation $\triangleleft \subseteq \mathcal{M} \times \mathcal{L}$ and verify the conditions $(\triangleleft 1)$, $(\triangleleft 2)$ and $(\triangleleft 3)$.

Instance: a pure region type system [BGH13]

- ▶ Take $(\mathcal{L}, \emptyset, \sqsubseteq, \sqcup) = (\text{Reg}, \perp, \leq, \vee)$ with

$$\text{return}_{\mathcal{L}}(r) = r \quad \text{bind}_{\mathcal{L}}(r, f) = f(r) \quad |r|_{\mathcal{L}} = \{r\}$$

$$\frac{\Gamma \vdash e_1 : r_1 \quad \Gamma, x:r_1 \vdash e_2 : r_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : r_2}$$

E.g. $\text{let } x = \text{if } \textit{cond} \text{ then } (\text{new}^{\ell_1} C) \text{ else } (\text{new}^{\ell_2} D) \text{ in } x : \text{CreatedAt}(\ell_1) \vee \text{CreatedAt}(\ell_2)$

- ▶ Work with the standard FJ operational semantics $(\mathcal{M} = \mathcal{V})$, and take $(v, h) \triangleleft r$ to be $(v, h) \vdash r$.

Instance: a Region-based Effect Type System [EHZ17]

- Take $\mathcal{L} = \text{Reg} \times \mathcal{P}(\Sigma^*)$ with the lattice structure defined componentwise

$e : (r, U)$ expresses that the result value of e is in region r and the generated event trace is in U .

- The monad functions are defined by

$$\text{return}_{\mathcal{L}}(r) = (r, \{\varepsilon\})$$

$$\text{bind}_{\mathcal{L}}((r, U), f) = (s, UV) \quad \text{where } (s, V) = f(r)$$

$$|(r, u)|_{\mathcal{L}} = \{r\}$$

The let-rule can be equivalently formulated as

$$\frac{\Gamma \vdash e_1 : (r_1, U_1) \quad \Gamma, x:r_1 \vdash e_2 : (r_2, U_2)}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : (r_2, U_1 U_2)}$$

E.g. $\text{let } x = \text{if } \text{cond} \text{ then } (\text{emit}(a); \text{new}^{\ell_1} C) \text{ else } (\text{new}^{\ell_2} D) \text{ in } \text{emit}(b); x$
has type $(\text{CreatedAt}(\ell_1) \vee \text{CreatedAt}(\ell_2), \{ab, b\})$.

- Work with the operational semantics with traces ($\mathcal{M} = \mathcal{V} \times \Sigma^*$),
and define $((v, h), w) \triangleleft (r, U) \Leftrightarrow ((v, h) \vdash r) \wedge (w \in U)$.

Instance: another Region-based Effect Type System [ESX21]

- Take \mathcal{L} to be the set of **finite partial functions** from Reg to $\mathcal{P}(\Sigma^*)$.

$e : r_1 \& U_1 \mid \cdots \mid r_n \& U_n$ expresses that the result of e is in region r_i and the trace is in U_i for some i .

- We need to define the lattice structure and the monad functions (omitted).

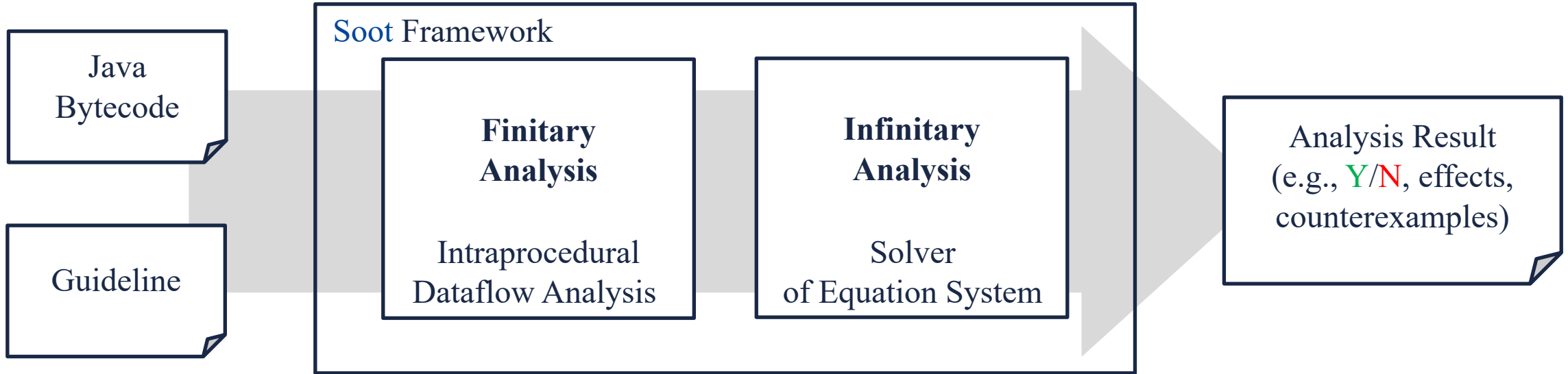
The let-rule can be equivalently formulated as

$$\frac{\Gamma \vdash e_1 : r_1 \& U_1 \mid \cdots \mid r_n \& U_n \quad \Gamma, x:r_i \vdash e_2 : T_i \text{ for } 1 \leq i \leq n}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \bigsqcup_{i=1}^n U_i \cdot T_i}$$

E.g. $\text{let } x = \text{if } \text{cond} \text{ then } (\text{emit}(a); \text{new}^{\ell_1} C) \text{ else } (\text{new}^{\ell_2} D) \text{ in } \text{emit}(b); x$
has type $\text{CreatedAt}(\ell_1) \& \{ab\} \mid \text{CreatedAt}(\ell_2) \& \{b\}$.

- Still work with the operational semantics with traces ($\mathcal{M} = \mathcal{V} \times \Sigma^*$),
but define $((v, h), w) \triangleleft (r_1 \& U_1 \mid \cdots \mid r_n \& U_n) \Leftrightarrow \exists i. ((v, h) \vdash r_i) \wedge (w \in U_i)$.
- Compared to [EHZ17], the above system is **more precise**.
But the cost is a **less efficient** type inference algorithm.

Prototype Implementation



A prototype implementation of type inference of [ESX21] based on the [Soot](#) framework:

- Effects are represented by the finitary abstraction based on the guideline automaton [HC14].
- The guideline also specifies the default effects of intrinsic functions.
- For libraries, we assume default effects or provide mockup code.

Summary

- ▶ We introduce a generic type system for FJ and prove a uniform soundness theorem.
- ▶ It unifies the systems investigated in the GuideForce project.
- ▶ The uniform framework is helpful when extending FJ to cover other language features:
Once $(\triangleleft 1)$ — $(\triangleleft 3)$ are verified, the soundness theorem is valid for the core FJ calculus.
We only need to prove the cases for the additional rules.

Thank you!

Comparing the Instances [EHZ17] and [ESX21]

Example: Suppose there are classes $D < C$ with two methods f and g .

Consider the class table:

<code>class C@CreatedAt (ℓ_1)</code>	<code>class D@CreatedAt (ℓ_2)</code>
<code> $f()$: Null & {a}</code>	<code> $f()$: Null & {b}</code>
<code> $g()$: Null & {aa}</code>	<code> $g()$: Null & {bb}</code>

- ▶ Let e be the FJ expression `if $cond$ then (new $^{\ell_1}$ C) else (new $^{\ell_2}$ D)`
- In [EHZ17], e has type `CreatedAt(ℓ_1) \vee CreatedAt(ℓ_2) & { ε }`
- In [ESX21], e has type `CreatedAt(ℓ_1) & { ε } | CreatedAt(ℓ_2) & { ε }`
- ▶ Consider expressions `let $x = e$ in $x.f()$; $x.f()$` and `let $x = e$ in $x.g()$`
- In [EHZ17], the former has type `null & { aa, ab, ba, bb }` and the latter has `null & { aa, bb }`
- In [ESX21], both have type `null & { aa, bb }`.
- ▶ The method g may have body `this. $f()$; this. $f()$` . Inlining loses precision in [EHZ17].
- ▶ [ESX21] is **more precise**, but the cost is a **less efficient** type inference algorithm.

Extension: Exception Handling

- ▶ Extend the syntax of FJ with expressions `throw e` and `try e_1 catch($C\ x$) e_2`
- ▶ For the operational semantics, work with e.g. $\mathcal{M} = \{N, E\} \times \mathcal{V}$
 - $(s, h) \vdash e \Downarrow N, v, h'$ means that e normalizes to v with the heap updated to h' .
 - $(s, h) \vdash e \Downarrow E, v, h'$ means that e throws an exception whose value is v and the heap is updated to h' .
- ▶ The monad functions are given by

$$\text{return}_{\mathcal{M}}(v, h) = (N, (v, h))$$

$$\text{bind}_{\mathcal{M}}((N, _), (x, (v, h))) = (x, (v, h))$$

$$\text{bind}_{\mathcal{M}}((E, (v, h)), _) = (E, (v, h))$$

$$|(_, (v, h))|_{\mathcal{M}} = (v, h).$$

- ▶ Think about all the possible cases of

$$\frac{(s, h) \vdash e_1 \Downarrow m_1 \quad (v_1, h_1) = |m_1|_{\mathcal{M}} \quad (s[x \mapsto v_1], h_1) \vdash e_2 \Downarrow m_2}{(s, h) \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow \text{bind}_{\mathcal{M}}(m_1, m_2)}$$

- ▶ Additional operational semantics rules for the new expressions such as

$$\frac{(s, h) \vdash e \Downarrow _, v, h'}{(s, h) \vdash \text{throw } e \Downarrow E, v, h'}$$

Extension: Exception Handling (cont.)

- Extend the pure region type system [BGH13] by taking $\mathcal{L} = \text{Reg} \times \text{Reg}$

$e : (r, s)$ says that e evaluates to a value in region r , or throws an exception whose value is in region s .

- The monad functions are define by

$$\text{return}_{\mathcal{L}}(r) = (r, \perp)$$

$$\text{bind}_{\mathcal{L}}((r, s), f) = (t, s \vee u) \quad \text{where } (t, u) = f(r)$$

$$|(r, s)|_{\mathcal{L}} = \{r\}$$

The let-rule can be equivalently formulated as

$$\text{LET} \frac{\Gamma \vdash e_1 : (r_1, s_1) \quad \Gamma, x:r_1 \vdash e_2 : (r_2, s_2)}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : (r_2, s_1 \vee s_2)}$$

- Additional typing rules for the new expressions such as

$$\text{THROW} \frac{\Gamma \vdash e : (r, s)}{\Gamma \vdash \text{throw } e : (\perp, r \vee s)}$$

- Lastly, define \triangleleft by $(N, (v, h)) \triangleleft (r, s) \Leftrightarrow (v, h) \vdash r$ and $(E, (v, h)) \triangleleft (r, s) \Leftrightarrow (v, h) \vdash s$

- Once $(\triangleleft 1)$ — $(\triangleleft 3)$ are verified, the soundness theorem is valid for the core FJ calculus, we only need to prove the cases for the additional rules.